A hybrid method for intuitionistic linguistic decision making with distance measure and aggregation operator

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Abstract

In this paper we propose a hybrid method for intuitionistic linguistic decision making and introduce the intuitionistic linguistic hybrid weighted distance (ILHWD) operator. It is a new aggregation that uses a unified model between distance measures and hybrid aggregation operator considering the importance degrees of both the individual distances and the attitudinal character of the decision maker. We study different families of the ILHWD operator. Finally, based on the presented operator, we develop a decision making approach and illustrate it with a numerical example under intuitionistic linguistic environment.

Keywords: Distance measures; hybrid weighted distance operator; intuitionistic linguistic set; decision making

1 Introduction

Because the objects are fuzzy and uncertain, the attributes involved in decision problems may not be always expressed as real numbers, and sometimes it is necessary to use another approach to deal with the uncertain information such as interval numbers [1], fuzzy set [2], intuitionistic fuzzy set (IFS) [3], linguistic information [4]. Among all these tools, the intuitionistic fuzzy set (IFS) proposed by Atanassov [3], considers not only a membership degree but also a non-membership degree, which is more appropriate to deal with the uncertainty and vagueness. Many decision making methods under the intuitionistic fuzzy setting have been developed [5-12]. Recently, based on intuitionistic fuzzy set and linguistic set, Wang and Li [13] proposed the concept of intuitionistic linguistic set (ILS), whose basic elements are intuitionistic linguistic values (ILV). The ILS can overcome the defects for intuitionistic fuzzy set which can only roughly represent criteria's membership and non-membership to a particular concept, such as ''good'' and ''bad'', etc., and for linguistic variables which usually implies that membership degree is 1, and the non-membership degree and hesitation degree of decision makers can not be expressed. Since its appearance, the ILS has been studied by a lot of authors [14-16].

Distance measures and aggregation operators are two useful tools for decision making. Recently, motivated by the idea of the ordered weighted averaging (OWA) operator [17], Merigó and Gil-Lafuente [18] introduced the ordered weighted averaging distance (OWAD) operator. Its main advantage is that it provides a parameterized family of distance aggregation operators between the maximum and the minimum distance. Su et al. [19] studied the use of intuitionistic linguistic sets in the OWAD operator, and develop two new intuitionistic linguistic aggregation operator: the intuitionistic linguistic weighted Hamming distance (ILWHD) operator and the intuitionistic linguistic OWAD (ILOWAD) operator. From the Ref. [19], we know that the ILWHD weights the given individual distances and takes only the importance of the given individual distances into consideration, while the ILOWAD operator considers only the attitudinal character of the decision maker and weights the ordered positions of the given individual distances instead of weighting the arguments themselves. Therefore, weights represent different aspects in both the ILWHD and ILOWAD operator. However, both the ILWHD and the ILOWAD operator considered only one of them. To solve this drawbacks, in this paper, we shall propose a new intuitionistic linguistic aggregation distance operator called the intuitionistic linguistic hybrid weighted distance (ILHWD) operator.

This rest of paper is organized as follows. In Section 2, we briefly review some basic concepts about intuitionistic linguistic set, the OWA operator, the OWAD operator and the ILOWAD operators. Section 3 presents the LHWD operator and analyzes a wide range of particular cases. Section 4 introduces a method based on the ILHWD for decision making problems. Section 5 summarizes the main conclusions found in the paper.

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2 Preliminaries

This section briefly reviews the intuitionistic linguistic set, the OWA operator, the OWAD and the ILOWAD operator.

2.1 THE LINGUISTIC APPROACH

The linguistic approach is an approximate technique, which represents qualitative aspects as linguistic values by means of linguistic variables. For computational convenience, let $S=\{s_\alpha | \alpha=0,1,...,l-1\}$ be a finite and totally ordered discrete term set, where l is the odd number and s_α represents a possible value for a linguistic variable. For example, when $l=9$, a set S could be given as follows:

$$S=\{s_0,s_1,s_2,s_3,s_4,s_5,s_6,s_7,s_8\}$$
$$=\{extremely\ poor, very\ poor, poor, slightly\ poor, fair, slightly\ good, good, very\ good, extremely\ good\}.$$

In these cases, it is usually required that there exist the following [20]:

1) A negation operator: $Neg(s_i)=s_{-i}$;
2) The set is ordered: $s_i \le s_j$ if and only if $i \le j$;
3) Maximum operator: $\max(s_i,s_j)=s_i$, if $i \ge j$;
4) Minimum operator: $\min(s_i,s_j)=s_i$, if $i \le j$.

In order to preserve all the given information, Xu[20] extended the discrete term set S to a continuous term set $\bar{S}=\{s_\alpha | \alpha\in[0,l]\}$, where, if $s_\alpha \in S$, then we call s_α the original term, otherwise, we call s_α the virtual term. In general, the decision maker uses the original linguistic terms to evaluate alternatives, and the virtual linguistic terms can only appear in the actual calculation [20].

Consider any two linguistic terms $s_\alpha, s_\beta \in \bar{S}$, and $\mu>0$, the operations are defined as follows [53]:

1) $s_\alpha \oplus s_\beta = s_{\alpha+\beta}$;
2) $\mu s_\alpha = s_{\mu\alpha}$;
3) $s_\alpha / s_\beta = s_{\alpha/\beta}$.

2.2 INTUITIONSTIC LINGUISTIC SET

Definition 1 ([13]). An ILS A in X is defined as

$$A=\left\{\left\langle x\left[h_{\theta(x)},(\mu_A(x),v_A(x))\right]\right\rangle \middle| x\in X\right\} \tag{1}$$

Here $h_{\theta(x)}\in\bar{S}$,and the $\mu_A(x)$ and $v_A(x)$ represent, respectively,the membership degree and non-membership degree of the element x to linguistic index $h_{\theta(x)}$, $0\le\mu_A(x)+v_A(x)\le 1$, for all $x\in X$.

For each ILS A in X, $\forall x\in X$, if

$$\pi_A(x)=1-\mu_A(x)-v_A(x), \tag{2}$$

then $\pi_A(x)$ is called the indeterminacy degree or hesitation degree of x to linguistic Index $h_{\theta(x)}$.

Definition 2 ([13]). Let $A=\left\{\left\langle x\left[h_{\theta(x)},(\mu_A(x),v_A(x))\right]\right\rangle \middle| x\in X\right\}$ be an ILS, the ternary group $\left\langle h_{\theta(x)},(\mu_A(x),v_A(x))\right\rangle$ is called an intuitionistic linguistic value (ILV), and A can also be viewed as a collection of the ILN. So, it can also be expressed as $A=\left\{\left\langle h_{\theta(x)},(\mu_A(x),v_A(x))\right\rangle \middle| x\in X\right\}$.

In addition, $\pi_A(x)=1-\mu_A(x)-v_A(x)$ represents the hesitancy degree, and it can also be called the intuitionistic linguistic fuzzy degree. For convenience, denote an ILV by $\tilde{a}=\left\langle s_{\theta(a)},(\mu(a),v(a))\right\rangle$, where $\mu(a),v(a)\ge 0$, $\mu(a)+v(a)\le 1$.

Let $\tilde{a}_1=\left\langle s_{\theta(a_1)},(\mu(a_1),v(a_1))\right\rangle$ and $\tilde{a}_2=\left\langle s_{\theta(a_2)},(\mu(a_2),v(a_2))\right\rangle$ be two intuitionistic linguistic values (ILVs), $\lambda\ge 0$,some operations of ILVs are defined as follows [13-14]:

1) $\tilde{a}_1+\tilde{a}_2=\left\langle s_{\theta(a_1)+\theta(a_2)},\left(1-(1-\mu(a_1))(1-\mu(a_2)),v(a_1)v(a_2)\right)\right\rangle$;
2) $\tilde{a}_1\otimes\tilde{a}_2=\left\langle s_{\theta(a_1)\times\theta(a_2)},\left(\mu(a_1)\mu(a_2),v(a_1)+v(a_2)-v(a_1)v(a_2)\right)\right\rangle$;
3) $\lambda\tilde{a}_1=\left\langle s_{\theta(a_1)^\lambda},\left(1-(1-\mu(a_1))^\lambda,(v(a_1))^\lambda\right)\right\rangle$.

Definition 3 ([13]). Let $\tilde{a}_1=\left\langle s_{\theta(a_1)},(\mu(a_1),v(a_1))\right\rangle$ be an ILV, the expected value $E(\tilde{a}_1)$ and score function $S(\tilde{a}_1)$ of an ILV $\tilde{a}_1$ can be represented as follows:

$$E(\tilde{a}_1)=s_{\theta(a_1)\times[\mu(a_1)+\frac{1}{2}(1-\mu(a_1)-v(a_1))]} \tag{3}$$

$$S(\tilde{a}_1)=\frac{\theta(a_1)}{l-1}\times\left[\mu(a_1)+\frac{1}{2}(1-\mu(a_1)-v(a_1))\right] \tag{4}$$

Definition 4 ([13]). Let $\tilde{a}_1=\left\langle s_{\theta(a_1)},(\mu(a_1),v(a_1))\right\rangle$ be an ILV, an accuracy function $H(\tilde{a}_1)$ of an ILV $\tilde{a}_1$ can be represented as follows:

$$H(\tilde{a}_1)=\frac{\theta(a_1)}{l-1}\times(\mu(a_1)+v(a_1)) \tag{5}$$

Definition 5 ([13]). If $\tilde{a}_1=\left\langle s_{\theta(a_1)},(\mu(a_1),v(a_1))\right\rangle$ and $\tilde{a}_2=\left\langle s_{\theta(a_2)},(\mu(a_2),v(a_2))\right\rangle$ are any two ILVs, then:

1) If $S(\tilde{a}_1)>S(\tilde{a}_2)$, then, $\tilde{a}_1>\tilde{a}_2$;
2) If $S(\tilde{a}_1)=S(\tilde{a}_2)$, then

   If $H(\tilde{a}_1)>H(\tilde{a}_2)$, then, $\tilde{a}_1>\tilde{a}_2$;

   If $H(\tilde{a}_1)=H(\tilde{a}_2)$, then, $\tilde{a}_1=\tilde{a}_2$.

Definition 6 ([13]). Let $\tilde{a}_1=\left\langle s_{\theta(a_1)},(\mu(a_1),v(a_1))\right\rangle$ and $\tilde{a}_2=\left\langle s_{\theta(a_2)},(\mu(a_2),v(a_2))\right\rangle$ be any two ILVs, then the distance between $\tilde{a}_1$ and $\tilde{a}_2$ is defined as follows:

$$d_{IL}(\tilde{a}_1,\tilde{a}_2)=\frac{1}{2(l-1)}\times \left(\left|\left(1+\mu(a_1)-v(a_1)\right)\theta(a_1)-\left(1+\mu(a_2)-v(a_2)\right)\theta(a_2)\right|\right) \quad (6)$$

2.3 THE OWA OPERATOR

The OWA operator [17] provides a parameterized family of aggregation operators that include the maximum, the minimum and the average criteria as special cases. This operator can be defined as follows:

Definition 7 An OWA operator of dimension n is a mapping OWA: $R^n \to R$ that has an associated weighting W with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$ such that:

$$OWA(a_1,a_2,...,a_n)=\sum_{j=1}^{n} w_j b_j \quad (7)$$

where b_j is the j th largest of the a_i.

2.4 THE OWAD OPERATOR

The OWAD (or Hamming OWAD) operator[18] is an extension of the traditional normalized Hamming distance by using the OWA operator. For two sets $A=\{a_1,a_2,...,a_n\}$ and $B=\{b_1,b_2,...,b_n\}$, the OWAD operator can be defined as follows:

Definition 8 An OWAD operator of dimension n is a mapping OWAD: $R^n \times R^n \to R$ that has an associated weighting W with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$ such that:

$$OWAD\left(\langle a_1,b_1\rangle,\langle a_2,b_2\rangle,...,\langle a_n,b_n\rangle\right)=\sum_{j=1}^{n} w_j d_j \quad (8)$$

where d_j is the j th largest of the $|a_i-b_i|$.

2.5 THE ILOWAD OPERATOR

Based on the above mentioned information, Su et al. [19] defined the ILWHD and the ILOWAD as follows:

Definition 9 Let $X=\{x_1,x_2,...,x_n\}$ be a finite universe of discourse, A and B be two Atanassov's intuitionistic linguistic sets in X, then

$$d(A,B)=\sum_{i=1}^{n} w_i d_{IL}(\tilde{a}_i,\tilde{b}_i) \quad (9)$$

is called the intuitionistic linguistic weighted Hamming distance (ILWHD) between A and B, where $w=(w_1,w_2,...,w_n)$ is the weight vector of $x_i (i=1,2,...,n)$, with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, and $\tilde{a}_i=\left\langle s_{\theta(a_i)},\left(\mu(a_i),v(a_i)\right)\right\rangle$ and $\tilde{b}_i=\left\langle s_{\theta(b_i)},\left(\mu(b_i),v(b_i)\right)\right\rangle$ are the i th ILV of A and B, respectively.

Definition 10 An ILOWAD operator of dimension n is a mapping ILOWAD: $\Omega^n \times \Omega^n \to R$ that has an associated weighting W with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$ such that:

$$ILOWAD\left((\tilde{a}_1,\tilde{b}_1),(\tilde{a}_2,\tilde{b}_2),...,(\tilde{a}_n,\tilde{b}_n)\right)=\sum_{j=1}^{n} w_j D_j \quad (11)$$

where Ω is the set of all ILV, D_j is the j th largest of the $d_{ILN}(\tilde{a}_i,\tilde{b}_i)$ value, $\tilde{a}_i=\left\langle s_{\theta(a_i)},\left(\mu(a_i),v(a_i)\right)\right\rangle$ and $\tilde{b}_i=\left\langle s_{\theta(b_i)},\left(\mu(b_i),v(b_i)\right)\right\rangle$ are the i th AILN of A and B, respectively.

From Definition 9 and 10, we know that the ILWHD weights the given individual distances while the ILOWAD operator weights the ordered positions of the given individual distances instead of weighting the arguments themselves. Therefore, weights represent different aspects in both the ILWHD and ILOWAD operator. However, both the of ILWHD and ILOWAD operator consider only one of them. To solve this drawback, in the following we shall propose the ILHWD operator.

3 ILHWD operator

Combining the advantage of the ILWHD measure and ILOWAD operator, we can develop the ILHWD operator as follows.

Definition 11 An ILHWD operator of dimension n is a mapping ILHWD: $\Omega^n \times \Omega^n \to R$ that has an associated weighting W with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$ such that:

$$ILHWD\left((\tilde{a}_1,\tilde{b}_1),(\tilde{a}_2,\tilde{b}_2),...,(\tilde{a}_n,\tilde{b}_n)\right)=\sum_{j=1}^{n} w_j \dot{D}_j \quad (11)$$

where $\dot{D}_j$ is the j th largest of the $\dot{d}_{IL}(\tilde{a}_i,\tilde{b}_i)$ (here $\dot{d}_{IL}(\tilde{a}_i,\tilde{b}_i)=n\omega_i d_{IL}(\tilde{a}_i,\tilde{b}_i)$, $i=1,2,...,n$), $\omega=(\omega_1,\omega_2,...,\omega_n)$ is the weighting vector of the $d_{IL}(\tilde{a}_i,\tilde{b}_i)$, with $\omega_i \in [0,1]$ and the sum of these weights is 1.

From a generalized perspective of the reordering step, we can distinguish between the descending ILHWD (DILHWD) operator and the ascending ILHWD (AILHWD) operator by using $w_j = w^*_{n-j+1}$, where w_j is the j th weight of the DILHWD and w^*_{n-j+1} the j th weight of the AILHWD operator.

Theorem 1 The ILWHD is a special case of the ILHWD operator.

Proof Let $w=\left(1/n,1/n,...,1/n\right)$, then

$$ILHWD = \sum_{j=1}^{n} w_j \dot{D}_j = \frac{1}{n}\sum_{j=1}^{n} \dot{d}_{IL}(\tilde{a}_i, \tilde{b}_i)$$
$$= \frac{1}{n}\sum_{j=1}^{n} n\omega_j n\omega_i d_{IL}(\tilde{a}_i, \tilde{b}_i) = \sum_{j=1}^{n} \omega_j d_{IL}(\tilde{a}_j, \tilde{b}_j)$$
$$= ILWHD$$

which completes the proof of Theorem 1.

Theorem 2 The ILOWAD operator is a special case of the ILHWD operator.

Proof Let $\omega = (1/n, 1/n, ..., 1/n)$, then

$$\dot{d}_{IL}(\tilde{a}_i, \tilde{b}_i) = d_{IL}(\tilde{a}_i, \tilde{b}_i), \ \ i = 1, 2, ..., n.$$

which completes the proof of Theorem 2.

From Definition 11 and the above theorems, we know that:

1) The ILHWD operator first weights the given individual distances, and then reorders the weighted arguments in descending order and weights these ordered individual distances by the ILHWD weights, and finally aggregates all the weighted arguments into a collective one.

2) The ILHWD operator generalizes both the ILWHD and ILOWAD operators, and reflects the importance degrees of both the given individual distances and their ordered positions.

By choosing a different manifestation of the weighting vector in the ILHWD operator, we are able to obtain different types of intuitionistic linguistic aggregation distance operators. The main advantage of using these particular cases is that we can select for each problem the particular case that we believe is closest to our interests.

Remark 1 For example, the max intuitionistic linguistic distance (MaxD), the min intuitionistic linguistic distance (MinD), the ILWHD and the ILOWAD are obtained as follows:

- The MaxD is found if $\omega = (1/n, 1/n, ..., 1/n)$, $w_1 = 1$ and $w_j = 0$, for all $j \neq 1$.
- The MinD is found if $\omega = (1/n, 1/n, ..., 1/n)$, $w_n = 1$ and $w_j = 0$, for all $j \neq n$
- More generally, if $w_k = 1$ and $w_j = 0$ for all $j \neq k$, we get the step-AILOWAD operator.
- The ILWHD is formed $w = (1/n, 1/n, ..., 1/n)$.
- The ILOWAD is obtained when $\omega = (1/n, 1/n, ..., 1/n)$.

Remark 2 Another particular case is the Olympic-ILWHD. This operator is found when $w_1 = w_n = 0$ and for all others $w_{j*} = 1/(n-2)$. Note that if $n = 3$ or $n = 4$, the olympic-ILWHD is transformed in the median- ILWHD.

Remark 3 Note that it is possible to present a general form of the olympic-ILWHD operator, considering that $w_j = 0$ for $j = 1, 2, ..., k, n, n-1, ..., n-k+1$, and for all others, $w_{j*} = 1/(n-2k)$ where $k < n/2$. Note that if $k = 1$, then this general form becomes the usual olympic-ILWHD. If $k = (n-1)/2$, then it becomes the median-ILWHD operator.

Remark 4 Additionally, it is also possible to present the contrary case of the general olympic-ILWHD operator. In this case, $w_j = 1/(2k)$ for $j = 1, 2, ..., k, n, n-1, ..., n-k+1$, and $w_j = 0$, for all others, where $k < n/2$. Note that if $k = 1$, then we get the contrary case of the median-ILWHD.

Remark 5 Using a similar method, we could develop numerous other families of I-IFOWAWA operators. For more information, refer to Ref. 6,18,19.

4 Decision making method with the ILHWD operator

The ILHWD operator is applicable in a wide range of situations such as in decision making, statistics, engineering and economics. In the following, we are going to develop an example in which we will see the applicability of the new approach. Assume a company that operates in Europe and North America is analyzing the general policy for the next year and they consider five possible strategies to follow (adopted from [19,21]): 1) A_1 = expand to the Asian market; 2) A_2 = expand to the African market; 3) A_3 = expand to the South American market; 4) A_4 = expand to all three continents; 5) A_5 = do not develop any expansion.

Depending on the situation, the expected benefits for the company will be different. The experts have considered five possible situations for the next year: 1) C_1 = negative-growth rate; 2) C_2 = growth rate near 0; 3) C_3 = low-growth rate; 4) C_4 = medium-growth rate; 5) C_5 = high-growth rate.

The group of experts of the company is constituted by three persons who give their own opinion about the expected results that may occur in the future, and the experts give the evaluation information by the intuitionistic linguistic variables using linguistic set $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8\}$. The expected results depending on the situation C_i and the alternative A_k are shown in Table 1-3. Note that the results are ILVs. According to the objectives of the decision-maker, each expert establishes his own ideal investment. The results are shown in Table 4.

TABLE 1 Intuitionistic linguistic payoff matrix-Expert 1.

	C_1	C_2	C_3	C_4	C_5
A_1	$\langle s_6,(0.5,0.4)\rangle$	$\langle s_4,(0.3,0.4)\rangle$	$\langle s_4,(0.6,0.3)\rangle$	$\langle s_3,(0.2,0.6)\rangle$	$\langle s_5,(0.4,0.4)\rangle$
A_2	$\langle s_2,(0.3,0.6)\rangle$	$\langle s_4,(0.5,0.4)\rangle$	$\langle s_6,(0.7,0.2)\rangle$	$\langle s_5,(0.5,0.5)\rangle$	$\langle s_2,(0.8,0.1)\rangle$
A_3	$\langle s_4,(0.2,0.7)\rangle$	$\langle s_7,(0.6,0.2)\rangle$	$\langle s_5,(0.6,0.3)\rangle$	$\langle s_2,(0.9,0.1)\rangle$	$\langle s_1,(0.4,0.4)\rangle$

A_4	$\langle s_5,(0.2,0.7)\rangle$	$\langle s_1,(0.5,0.5)\rangle$	$\langle s_2,(0.3,0.6)\rangle$	$\langle s_2,(0.5,0.4)\rangle$	$\langle s_5,(0$
A_5	$\langle s_5,(0.7,0.2)\rangle$	$\langle s_5,(0.2,0.8)\rangle$	$\langle s_1,(0.1,0.9)\rangle$	$\langle s_3,(0.3,0.6)\rangle$	$\langle s_5,(0.4,0.4)\rangle$

TABLE 2 Intuitionistic linguistic payoff matrix-Expert 2.

	C_1	C_2	C_3	C_4	C
A_1	$\langle s_5,(0.7,0.2)\rangle$	$\langle s_6,(0.8,0)\rangle$	$\langle s_5,(0.6,0.3)\rangle$	$\langle s_7,(0.5,0.5)\rangle$	$\langle s_3,(0.$
A_2	$\langle s_2,(0.2,0.7)\rangle$	$\langle s_1,(0.4,0.6)\rangle$	$\langle s_5,(0.7,0.2)\rangle$	$\langle s_3,(0.6,0.4)\rangle$	$\langle s_6,(0.$
A_3	$\langle s_4,(0.4,0.5)\rangle$	$\langle s_5,(0.7,0.3)\rangle$	$\langle s_2,(0.5,0.3)\rangle$	$\langle s_5,(0.2,0.7)\rangle$	$\langle s_5,(0.$
A_4	$\langle s_2,(0.2,0.8)\rangle$	$\langle s_4,(0.7,0.2)\rangle$	$\langle s_5,(0.2,0.7)\rangle$	$\langle s_6,(0.5,0.4)\rangle$	$\langle s_3,(0.$
A_5	$\langle s_1,(0.4,0.5)\rangle$	$\langle s_5,(0.9,0)\rangle$	$\langle s_4,(0.4,0.5)\rangle$	$\langle s_7,(0.8,0.1)\rangle$	$\langle s_2,(0.4,0.4)\rangle$

TABLE 3 Intuitionistic linguistic payoff matrix-Expert 3.

	C_1	C_2	C_3	C_4	C_5
A_1	$\langle s_5,(0.3,0.6)\rangle$	$\langle s_6,(0.4,0.5)\rangle$	$\langle s_7,(0.3,0.6)\rangle$	$\langle s_7,(0.8,0.1)\rangle$	$\langle s_5,(0.8,0.2)\rangle$
A_2	$\langle s_2,(0.3,0.7)\rangle$	$\langle s_3,(0.5,0.4)\rangle$	$\langle s_7,(0.8,0.1)\rangle$	$\langle s_1,(0.3,0.6)\rangle$	$\langle s_3,(0.4,0.5)\rangle$
A_3	$\langle s_7,(0.8,0.2)\rangle$	$\langle s_5,(0.4,0.6)\rangle$	$\langle s_6,(0.5,0.4)\rangle$	$\langle s_2,(0.2,0.7)\rangle$	$\langle s_6,(0.8,0.2)\rangle$
A_4	$\langle s_1,(0.4,0.5)\rangle$	$\langle s_5,(0.7,0.2)\rangle$	$\langle s_6,(0.5,0.4)\rangle$	$\langle s_4,(0.6,0.3)\rangle$	$\langle s_6,(0$
A_5	$\langle s_3,(0.7,0.2)\rangle$	$\langle s_4,(0.4,0.6)\rangle$	$\langle s_6,(0.7,0.2)\rangle$	$\langle s_6,(0.6,0.2)\rangle$	$\langle s_5,(0$

TABLE 4 Ideal strategy.

	C_1	C_2	C_3	C_4	C
e_1	$\langle s_8,(0.8,0.1)\rangle$	$\langle s_7,(0.8,0.1)\rangle$	$\langle s_7,(0.9,0.1)\rangle$	$\langle s_7,(1,0)\rangle$	$\langle s_7,(0.9,0)\rangle$
e_2	$\langle s_7,(0.9,0.1)\rangle$	$\langle s_7,(1,0)\rangle$	$\langle s_7,(0.8,0.1)\rangle$	$\langle s_8,(0.9,0.1)\rangle$	$\langle s_7,(0.8,0.1)\rangle$
e_3	$\langle s_7,(0.8,0.1)\rangle$	$\langle s_7,(0.9,0.1)\rangle$	$\langle s_8,(1,0)\rangle$	$\langle s_8,(1,0)\rangle$	$\langle s_7,(0.9,0.1)\rangle$

With this information, we can aggregate the available information in order to make a decision. First, we aggregate the information of the three experts to obtain a unified payoff matrix represented in the form of individual distances between the available and ideal alternatives. We use the intuitionistic linguistic weighted averaging (ILWA) operator [13] to obtain this matrix assuming that $V=(0.3,0.3,0.4)$. The results are shown in Table 5.

It is now possible to develop different methods based on the ILHWD operator in order to make a decision. In this example, we consider the MaxD, the MinD, the ILWHD and the ILOWAD. We assume the following weighting vector $W=(0.1,0.1,0.2,0.2,0.4)$ and $\omega=(0.11,0.24,0.30,0.24,0.11)$.

TABLE 5 Collective results in the form of individual distances.

	C_1	C_2	C_3	C_4	C_5
A_1	0.411	0.475	0.299	0.438	0.411
A_2	0.672	0.461	0.251	0.345	0.499
A_3	0.362	0.239	0.308	0.558	0.419
A_4	0.663	0.33	0.514	0.412	0.586
A_5	0.458	0.44	0.365	0.292	0.484

The results are shown in Table 6.

TABLE 6 Aggregated Results.

	MaxD	MinD	ILWHD	ILOWAD	ILHWD
A_1	0.475	0.299	0.400	0.375	0.382
A_2	0.672	0.251	0.432	0.379	0.324
A_3	0.558	0.239	0.401	0.327	0.363
A_4	0.663	0.33	0.519	0.442	0.421
A_5	0.484	0.292	0.415	0.372	0.356

A further interesting issue is to establish an ordering of the alternatives. This becomes useful when we want to consider more than one alternative. The results are shown in Table 7. As we can see, depending on the aggregation operator used, the ordering of the investment strategies may be different.

TABLE 7 Ordering of the Strategies

	Ordering
MaxD	$A_1 \succ A_5 \succ A_3 \succ A_4 \succ A_2$
MinD	$A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4$
ILWHD	$A_1 \succ A_3 \succ A_5 \succ A_2 \succ A_4$
ILOWAD	$A_3 \succ A_5 \succ A_1 \succ A_2 \succ A_4$
ILHWD	$A_2 \succ A_5 \succ A_3 \succ A_1 \succ A_4$

5 Conclusions

In this paper, we have presented the ILHWD operator. The main advantage of this operator is that it is generalizes both the ILWHD and ILOWAD operators, and reflects the importance degrees of both the individual distances and the attitudinal character of the decision maker. We have analyzed its application in a group decision making problem regarding the selection of investments. We have seen that this approach provides better information for decision-making because it is able to consider a wide range of scenarios depending on the interests of the decision-maker. We have also seen that, depending on the particular type of aggregation operator used, the results may lead to different decisions.

In future research, we expect to develop further extensions of this approach by using other extensions such as the use of unified aggregation operators, more complex structures and applying it to different decision making problems such as in financial and production management.

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